

# Short Papers

## Feedback in Distributed Amplifiers

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**Abstract**—Parallel and series feedback effects are described for application in distributed amplifiers, and their frequency dependence characteristics are discussed. A six-element distributed amplifier utilizing series feedback is described. This amplifier demonstrates a 2 dB improvement in small-signal gain over the 2 to 18 GHz frequency range compared to a regular distributed amplifier using the same active device.

### I. INTRODUCTION

A distributed amplifier can generally be regarded as a string of three-terminal (two-port) elements with gain, connected together to create input and output artificial transmission lines [1]. The signal is a decaying wave on the input transmission line, and a growing wave on the output transmission line. At low frequencies the decay rate is low on the gate line, and the growth rate is high on the drain line. At high frequencies the reverse is true, leading to lower gain. However, more signal is dissipated in the input and output line terminations at low frequencies, which helps flatten the gain slope of the amplifier. At the high-frequency end of the amplifier's operation band  $f_c$ , its behavior is analogous to a 100 percent directional coupler with gain. At this frequency in a properly designed amplifier the power dissipated in either termination is negligible and the gain (or power in the case of a power amplifier) is maximized [2]. The gain of the amplifier at this frequency is limited by the maximum available gain ( $G_{max}$ ) of the individual constituent gain elements. Therefore, the maximum achievable flat gain over the operation band is  $G_{max}$  at  $f_c$ . Among the various factors affecting  $G_{max}$  are the input and output loss factors of the gain element. The input loss factor determines the rate at which the input signal decays along the line, and the output loss factor affects the growth rate of the signal along the output line. The optimum number of gain elements to be used in the distributed amplifier is also determined by the input and output loss factors of the elements [3]. Through the use of feedback in the gain element, the input and output losses can be changed to achieve higher values of  $G_{max}$  and amplifier gain. The issue of stability will be discussed separately.

### II. A SIMPLE MODEL

In the common-source configuration of an FET any element separating the source from ground gives rise to feedback known as series feedback. It is distinct from parallel feedback, which is achieved with external elements connecting gate and drain terminals. Examples of both schemes will be given here with an emphasis on series feedback. The goal is to reduce losses in the input and output circuits.

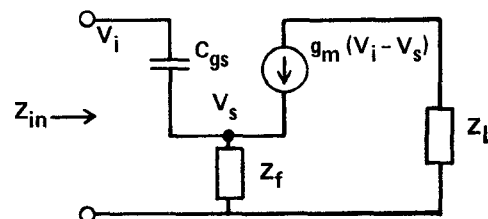


Fig. 1. A simple model to demonstrate the broad-band nature of a series feedback scheme.

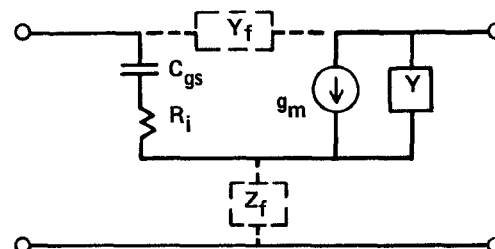


Fig. 2. The general model used to evaluate parallel and series feedback

Consider the very simple FET model shown in Fig. 1. Output conductance and input resistance are neglected. A series element  $Z_f$  is introduced in the source circuit whose magnitude and phase determine the loss as seen from the input. The input impedance of this configuration is given by

$$Z_{in} = \frac{1}{j\omega C_{gs}} + Z_f + \frac{Z_f g_m}{j\omega C_{gs}} \quad (1)$$

independent of the load impedance  $Z_L$ . The positive real part of  $Z_{in}$  represents input loss. As can be seen, inductive or resistive components of  $Z_f$  contribute to loss, while a capacitive  $Z_f$  gives rise to a negative resistance at the input. It is therefore possible to use a capacitive element to control the level of input loss caused by a resistive element in  $Z_f$  (or by other elements in a more general model). In the simple case of Fig. 1, a parallel RC combination with a time constant of

$$RC = C_{gs} / g_m \quad (2)$$

results in zero input loss at all frequencies. The point to be emphasized is the broad-band nature of this loss compensation. It will be seen that similar broad-band effects can be achieved with more complete models as well.

### III. A MORE GENERAL APPROACH

Consider the unilateral FET model of Fig. 2. Series and parallel feedback elements are shown in dotted lines. In order to obtain the overall impedance matrix for the FET with series feedback,

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the impedance of the feedback element  $Z_f = (\rho_{fb} + j\xi_{fb})$  is added to every element of the impedance matrix of the unilateral FET. Similarly, for parallel feedback, the admittance of the feedback element  $Y_f = (\rho_{fb} + j\xi_{fb})$  is added to the admittance matrix elements of the unilateral FET.

For any given impedance or admittance matrix (immittance), the maximum available gain of the two-port is given by [4]

$$G_{\max} = \frac{|\gamma_{21}|^2}{2\rho_{11}\rho_{22} - \text{Re}(\gamma_{12}\gamma_{21}) + \{[2\rho_{11}\rho_{22} - \text{Re}(\gamma_{12}\gamma_{21})]^2 - |\gamma_{12}\gamma_{21}|^2\}^{1/2}} \quad (3)$$

where  $\rho_{ij}$  is the real part and  $\xi_{ij}$  is the imaginary part of the  $ij$ th immittance matrix element  $\gamma_{ij}$ .

Since for a unilateral FET  $\gamma_{12} = 0$ , equation (3) reduces to

$$G_{\max}^u = \frac{|\gamma_{21}|^2}{2\rho_{11}\rho_{22}} \quad (4)$$

When the feedback term is added to the FET it will not stay

$$\Delta G_{\max} = -R_f g_m \frac{2R_f \omega^2 C_{gs} C_{ds} (C_{ds} g_m - 2C_{gs} G_{ds}) + 2g_m C_{gs} G_{ds} (1 + G_{ds} R_i) - g_m^2 C_{ds}}{8\omega^2 C_{gs}^3 (G_{ds} R_i)^2} \quad (10)$$

unilateral. For studying the effects of various feedback elements on  $G_{\max}$  a perturbation approach is taken where the feedback is considered small and only the first-order correction term  $\Delta G_{\max}$  to (4) is considered, i.e.,

$$G_{\max} = G_{\max}^u + \Delta G_{\max} + \dots \quad (5)$$

If the feedback element is resistive ( $\rho_{fb}$ ), the correction term to unilateral gain, which is the first-order expansion term of (3) about  $\rho_{fb} = 0$ , is given by

$$\Delta G_{\max} = \rho_{fb} \frac{4\rho_{11}\rho_{21}\rho_{22} + |\gamma_{21}|^2(\rho_{21} - 2\rho_{11} - 2\rho_{22})}{8\rho_{11}^2\rho_{22}^2} \quad (6)$$

Note that  $\rho_{fb}$  has dimensions of resistance for series feedback and dimensions of conductance for parallel feedback. The approximation is valid if  $\rho_{fb} \ll (\rho_{11}\rho_{22})^{1/2}$ .

Similarly, for reactive feedback elements ( $\xi_{fb}$ ) the correction term is found to be

$$\Delta G_{\max} = \xi_{fb} \frac{4\rho_{11}\xi_{21}\rho_{22} - |\gamma_{21}|^2\xi_{21}}{8\rho_{11}^2\rho_{22}^2} \quad (7)$$

Equations (6) and (7) need to be evaluated for the two configurations of parallel and series feedback. For series feedback the impedance matrix elements for the unilateral FET will be used to substitute  $\gamma_{ij}$ , and the feedback terms ( $\rho_{fb}$ ) and ( $\xi_{fb}$ ) will have dimensions of impedance. For parallel feedback, admittance matrix elements will be substituted for  $\gamma_{ij}$ , and the feedback terms will have admittance dimensions.

Referring to Fig. 2 the impedance matrix of the unilateral FET is found to be

$$[Z] = \begin{bmatrix} \frac{1 + j\omega R_i C_{gs}}{j\omega C_{gs}} & 0 \\ -\frac{g_m}{j\omega C_{gs} Y} & \frac{1}{Y} \end{bmatrix} \quad (8)$$

The corresponding admittance matrix is

$$[Y] = \begin{bmatrix} \frac{j\omega C_{gs}}{1 + j\omega R_i C_{gs}} & 0 \\ -\frac{g_m}{1 + j\omega R_i C_{gs}} & Y \end{bmatrix} \quad (9)$$

Using these matrices to evaluate (6) and (7),  $\Delta G_{\max}$  will be evaluated for series and parallel feedback in the next sections.

#### IV. SERIES FEEDBACK

For series feedback, matrix elements of (8) are substituted into (6) and (7) to find the effect of resistive and reactive feedback elements on  $G_{\max}$ . The results are

for resistive feedback, and

$$\Delta G_{\max} = \xi_f g_m \frac{4C_{gs}^2 \omega^2 G_{ds} R_i - g_m^2}{8C_{gs}^3 \omega^3 G_{ds} R_i^2} \quad (11)$$

for reactive feedback. Here,  $\xi_f = \omega L_f$  if the feedback element is an inductor, and  $\xi_f = -1/\omega C_f$  if it is a capacitor. It can be seen from (11) that capacitive series feedback increases  $G_{\max}$  at low frequencies; i.e., it acts as positive feedback. The reverse is true for inductive series feedback. For frequencies higher than a transition frequency given by

$$\omega_t = \frac{g_m}{2C_{gs}\sqrt{G_{ds} R_i}} \quad (12)$$

capacitive feedback becomes negative and inductive feedback becomes positive. In the case of resistive feedback the picture is more involved. The role of the resistor as a positive or negative feedback element is not only frequency dependent but also, at a given frequency, dependent on various FET parameters. A transition frequency still exists, given by

$$\omega_t^2 = g_m \frac{g_m C_{ds} - 2C_{gs} G_{ds} (1 + G_{ds} R_i)}{2C_{gs} C_{ds} R_i (g_m C_{ds} - 2C_{gs} G_{ds})} \quad (13)$$

But below this transition frequency the feedback can be either positive or negative, depending on the sign of the numerator in (13). Specifically, series source resistance in an FET acts as a negative feedback element only if the numerator of (13) is negative. If the denominator of (13) is also negative, then a transition frequency exists above which the same resistive element acts as positive feedback. Both of these conditions are met if

$$g_m < 2G_{ds} \frac{C_{gs}}{C_{ds}}$$

At low frequencies below the transition point, the increase in gain due to capacitive feedback and the reduction in gain due to resistive feedback do not have the same frequency dependence. Therefore a series RC element cannot be used for broad-band loss

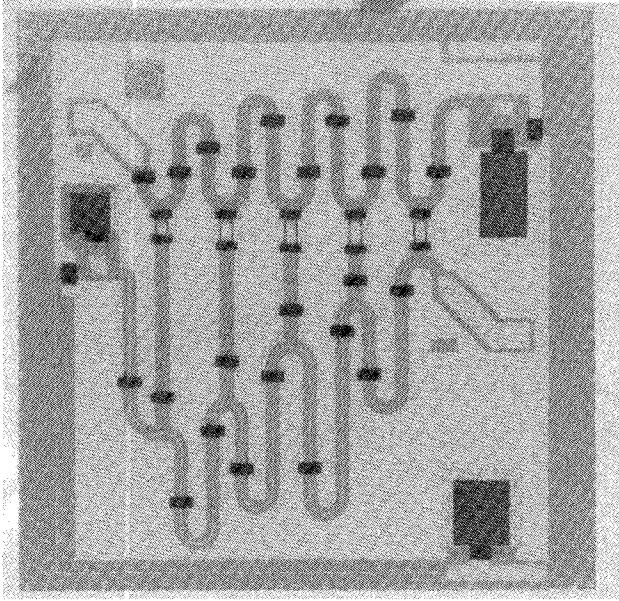


Fig. 3. Conventional distributed amplifier with CPW transmission lines.

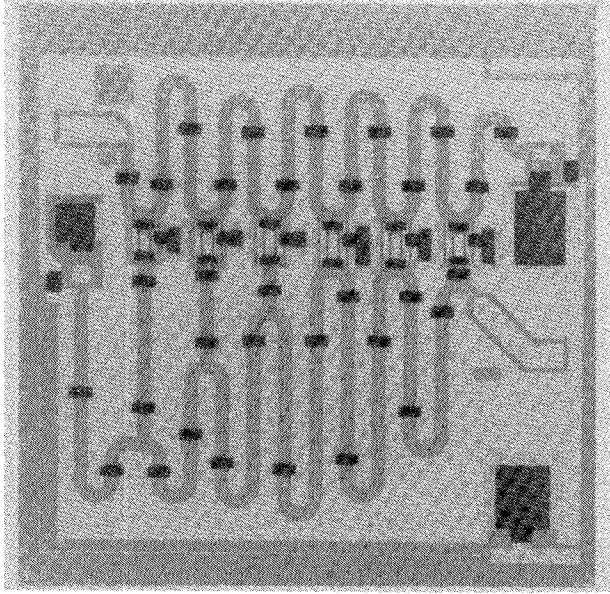


Fig. 4. Distributed amplifier employing parallel RC series feedback.

compensation. On the other hand if the feedback element is a parallel RC combination, then  $R_f$  has to be replaced with  $R_f/[1+(\omega R_f C_f)^2]$  and  $\xi_f$  with  $-\omega R_f^2 C_f/[1+(\omega R_f C_f)^2]$ . Depending on the values of  $R_f$  and  $C_f$ , the same frequency dependence can be obtained for  $\Delta G_{\max}$ , leading to the possibility of broad-band loss compensation.

## V. PARALLEL FEEDBACK

In order to find the first-order effect of parallel feedback elements on  $G_{\max}$  of the FET, matrix elements of (9) are substituted into (6) and (7). The following result is obtained for resistive parallel feedback:

$$\Delta G_{\max} = -G_f \frac{g_m^3 + 2g_m^2(C_{gs}^2\omega^2 G_{ds} R_i^2 + C_{gs}^2\omega^2 R_i + G_{ds}) + 4g_m C_{gs}^2\omega^2 G_{ds} R_i}{2(C_{gs}^2\omega^2 G_{ds} R_i)^2} \quad (14)$$

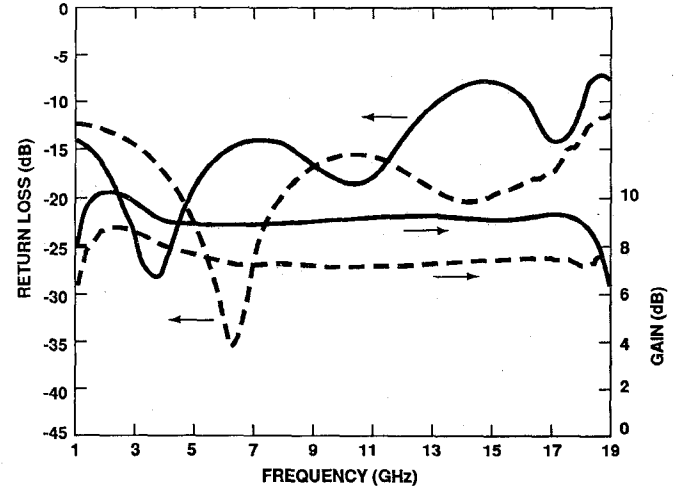


Fig. 5. Measured gain and return conventional (dotted) and feedback (solid) distributed amplifiers.

For reactive parallel feedback the corresponding result is

$$\Delta G_{\max} = \xi_f g_m \frac{4C_{gs}^2\omega^2 G_{ds} R_i - g_m^2}{8C_{gs}^3\omega^3 G_{ds}^2 R_i} \quad (15)$$

where  $\xi_f = \omega C_f$  for capacitive feedback, and  $\xi_f = -1/\omega L_f$  for inductive feedback.

It can be seen from (14) that a parallel resistive element always gives negative feedback reducing  $G_{\max}$ . This is true regardless of frequency or FET parameters. From (15) it can be seen that for reactive feedback elements a transition frequency exists which is the same as the one given in (12) for series feedback. The difference, however, is the fact that the role of capacitive and inductive elements in giving positive or negative feedback is reversed.

## VI. EXPERIMENTAL RESULTS

In Section IV it was shown that a broad-band method of loss control in an FET involves using the parallel RC combination as a series feedback element. This feedback scheme was chosen to improve the gain of a distributed amplifier. An existing distributed amplifier at the time used half-micron optical gate FET's in a conventional design to yield a gain of 7 dB over the frequency range of 2 to 18 GHz [5]. The same active device was used in the design of the feedback distributed amplifier in order to boost the gain from 7 to 9 dB. The source was grounded through a parallel RC combination ( $R = 8 \Omega$ ,  $C = 0.7$  pF for a 160- $\mu\text{m}$ -wide device). In both cases the total FET periphery was optimized for maximum flat gain over the band. The design approach for the distributed amplifier was not altered with the use of feedback. The loss compensated FET was simply treated as a different and higher gain element. The maximum available gain of the active device with feedback in this case was on the average 2.0 dB higher than that of the FET alone. This was mainly due to the fact that the magnitude of  $S_{11}$  (and  $S_{22}$  to a lesser extent) became closer to unity, thereby reducing the associated losses. Since input line loss was reduced by feedback, the optimum FET periphery increased from 710  $\mu\text{m}$  to 1015  $\mu\text{m}$ . Figs. 3 and 4 show the regular and feedback distributed ampli-

fiers, both using coplanar waveguide transmission lines. Measured gain and return loss data for both amplifiers are shown in Fig. 5.

## VII. STABILITY AND SENSITIVITY OF THE AMPLIFIER

A distributed amplifier is generally least stable in the vicinity of its cutoff frequency  $f_c$ . The addition of positive feedback further destabilizes the amplifier. Therefore the maximum level of positive feedback usable in a given amplifier is limited by the requirement of unconditional stability of the amplifier near its cutoff frequency. It is possible to check for this requirement at the design stage. Another side effect of positive feedback is that it tends to make the amplifier design more sensitive to parameter variations. Distributed amplifiers are known for their general lack of sensitivity to processing parameter variations, resulting in their high processing yield. With positive feedback the high yield can be traded off with amplifier gain.

## VIII. CONCLUSION

Simple expressions were derived giving first-order changes in maximum available gain for an FET with various parallel and series feedback elements. Transition frequencies were established showing that the effect of feedback elements on the stability of an active device can change sign over frequency. It was also shown that the parallel  $RC$  combination used as series feedback in an FET has a wide-band effect on the available gain of the device. This effect was used in the design of a 2 to 18 GHz distributed amplifier to demonstrate the increased gain that can be achieved over the entire frequency band of the amplifier.

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## Enhanced Through-Reflect-Line Characterization of Two-Port Measuring Systems Using Free-Space Capacitance Calculation

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**Abstract**—A through-reflect-line calibration procedure is presented wherein the free-space capacitance and propagation factor of the line standard are used to determine the line characteristic impedance. The method is applied to measurement of a microstrip via.

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## I. INTRODUCTION

At RF and microwave frequencies the calibration of both vector automatic network analyzers (ANA's) and test fixturing is required to accurately determine the scattering parameters of a device under test (DUT). The most frequently used calibration procedure for noncoaxial measurements is the TRL (through-reflect-line) method [1], which uses as standards a through connection, an arbitrary reflection, and a transmission line of known length and known characteristic impedance  $Z_0$ . An approximate determination of  $Z_0$  can be made using time-domain reflectometry (TDR); however, TDR cannot determine frequency variations of  $Z_0$ . For example, microstrip  $Z_0$  can vary by 5 percent from dc to 10 GHz for a typical line [2]. In this paper we present a method of microstrip TRL calibration that uses the calculated free-space capacitance and an experimentally determined propagation constant to determine the frequency-dependent complex characteristic impedance. This impedance is used in the conventional TRL algorithm so that our method is designated enhanced TRL (ETRL).

## II. DEVELOPMENT OF THE METHOD

For uniform TEM transmission lines the characteristic impedance is related to the free-space capacitance and the effective dielectric constant by [3]

$$Z_c = \frac{Z_0}{\sqrt{\epsilon_e}} \quad (1)$$

with

$$Z_0 = \frac{1}{C_0 c} \quad (2)$$

where  $Z_c$  is the dielectric-loaded characteristic impedance of the line,  $Z_0$  is its free-space characteristic impedance,  $C_0$  is the free-space capacitance per unit length of the line,  $c$  is the velocity of light, and  $\epsilon_e$  is the effective dielectric constant.  $Z_c$  is also related to the propagation constant  $\gamma$  by [4]

$$\epsilon_e = \frac{-\gamma^2 c^2}{\omega^2} \quad (3)$$

where  $\gamma = \alpha + j\beta$  is available from measurement as a by-product of the TRL algorithm [5]:

$$\gamma = \frac{\ln \left( \frac{A \pm \sqrt{A^2 - 4}}{2} \right)}{l} \quad (4)$$

Here  $A = T_{11} \cdot T_{22} + T_{11} \cdot T_{22} - T_{21} \cdot T_{12} - T_{12} \cdot T_{21}$ ,  $T_{ij}$  and  $T_{ji}$  are the chain scattering parameters [5] of the line and through calibration standards respectively, and  $l$  is the length of the line standard.

Combining (1)–(3) and taking the negative root,

$$Z_c = \frac{j\omega}{c^2 C_0 \gamma} \quad (5)$$

which is used in the standard TRL algorithm as follows. At each frequency, all measurements are transformed from the measurement reference impedance to  $Z_c$  so that the inserted line becomes reflectionless. Application of TRL then determines the  $S$  parameters of the error network references to  $Z_c$ . To complete calibration the  $S$  parameters of the error network are returned to the measurement reference impedance system (usually 50  $\Omega$ ).